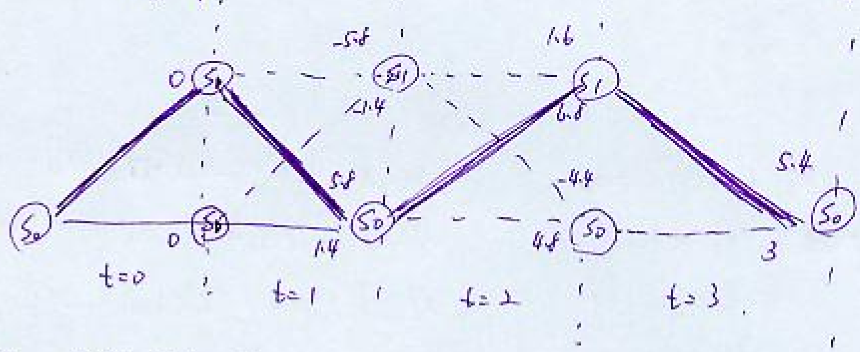
HW 1



1. (a) SOVA:

① $t=0$

$S_1: M^*([r|V=1]_1) = 0 + 2(-1.6+1.6) + 0 = 0$

$S_0: M^*([r|V=-1]_1) = 0 + 2(1.6-1.6) + 0 = 0$

② $t=1$

$S_1: M^*([r|V=1]_2) = M^*_1([r|V=-1]_1) + 2(1.1-1.8) = -1.4$

$M^*([r|V=-1]_2) = 0 + 2(-1.1-1.8) = -5.8$

$S_0: M^*([r|V=1]_2) = 2(1.1+1.8) = 5.8$

$M^*([r|V=-1]_2) = 2(-1.1+1.8) = 1.4$

Survivor of S_1 : -1.4

Survivor of S_0 : 5.8

③ $t=2$

$S_1: M^*([r|V=1]_3) = 5.8 + 2(-0.5+1) = 6.8$

$M^*([r|V=-1]_3) = -1.4 + 2(0.5+1) = 1.6$

$S_0: M^*([r|V=1]_3) = -1.4 + 2(-0.5-1) = -4.4$

$M^*([r|V=-1]_3) = 5.8 + 2(0.5-1) = 4.8$

Survivor of $S_1 = 6.8$

Survivor of S_0 : 4.8.

④ $t=3$

$S_0.\ M^*([r|V=1]_4) = 6.8 + 2(0.1-0.8) = 5.4$

$M^*([r|V=-1]_4) = 4.8 + 2(-0.1-0.8) = 3$

Survivor : $M^*([r|V=1]_4) = 5.4$

Survivor path show above

From survivor path, we can decode information bit : $(1, 1, 1, 1) = (u_0, u_1, u_2, u_3)$

the log likelihood ratio of u_0 ($L(u_0)$) is $1 \cdot \min(\Delta_3, \Delta_2, \Delta_1, \Delta_0) = 1 \cdot \min(X, \frac{1}{2}(6.8-1.6), \frac{1}{2}(5.8-1.4),) = 2.2$

$L(u_1) = 1 \cdot \min(\Delta_3, \Delta_2, \Delta_1) = 1 \cdot \min(X, X, \frac{1}{2}(5.8-1.4)) = 2.2$

$L(u_2) = 1 \cdot \min(\Delta_3, \Delta_2) = 1 \cdot \min(X, \frac{1}{2}(6.8-1.6)) = 2.6$

$L(u_3) = 1 \cdot \min(\Delta_3) = 1 \cdot \frac{1}{2}(5.4-3) = 1.2$

(b) Max - log - MAP :

先算 Log - MAP:

(c) γ:

$\gamma_0^*(s_0 \to s_0) = -1.6 \times (-1) + 1.6 \times (-1) = 0$

$\gamma_0^*(s_0 \to s_1) = -1.6 \times 1 + 1.6 \times 1 = 0$.

$\gamma_1^*(s_1 \to s_1) = 1.1 \times (-1) + (-1.8) \times 1 = -2.9$

$\gamma_1^*(s_1 \to s_0) = 1.1 \times 1 + (-1.8) \times (-1) = 2.9$

$\gamma_1^*(s_0 \to s_0) = (1.1) \times (-1) + (-1.8) \times (-1) = 0.7$

$\gamma_1^*(s_0 \to s_1) = (1.1) \times 1 + (-1.8) \times 1 = -0.7$

$\gamma_2^*(s_0 \to s_0) = -0.5 \times (-1) + 1 \times (-1) = -0.5$

$\gamma_2^*(s_0 \to s_1) = -0.5 \times 1 + 1 \times 1 = 0.5$

$\gamma_2^*(s_1 \to s_1) = -0.5 \times (-1) + 1 \times 1 = 1.5$

$\gamma_2^*(s_1 \to s_0) = -0.5 \times 1 + 1 \times (-1) = -1.5$

$\gamma_3^*(s_1 \to s_0) = 0.1 \times 1 + 0.8 \times (-1) = -0.7$

$\gamma_3^*(s_0 \to s_0) = 0.1 \times (-1) + 0.8 \times (-1) = -0.9$

α:

$\alpha_1^*(s_0) = [\gamma_0^*(s_0 \to s_1) + \alpha_0^*(s_0)] = 0 + 0 = 0$

$\alpha_1^*(s_1) = [\gamma_0^*(s_0 \to s_1) + \alpha_0^*(s_0) = 0 + 0 = 0$

$\alpha_2^*(s_0) = \max\left([\gamma_1^*(s_0 \to s_0) + \alpha_1^*(s_0)], [\gamma_1^*(s_1 \to s_0) + \alpha_1^*(s_1)] \right) = 3$

$\alpha_2^*(s_1) = \max\left([\gamma_1^*(s_0 \to s_1) + \alpha_1^*(s_0)], [\gamma_1^*(s_1 \to s_1) + \alpha_1^*(s_1)] \right) = -0.67$

$\alpha_3^*(s_0) = 2.52$, $\alpha_3^*(s_1) = 3.582$

β:

$\beta_4^*(s_0) = \beta_4^*(s_1) = 0$

$\beta_3^*(s_0) = [\gamma_3^*(s_0 \to s_0) + \beta_4^*(s_0)] = -0.9$

$\beta_3^*(s_1) = [\gamma_3^*(s_1 \to s_0) + \beta_4^*(s_0)] = -0.7$

$\beta_2^*(s_0) = \max\left([\gamma_2^*(s_0 \to s_1) + \beta_3^*(s_1)], [\gamma_2^*(s_0 \to s_0) + \beta_3^*(s_0)] \right) = 0.0632$

$\beta_2^*(s_1) = \max\left([\gamma_2^*(s_1 \to s_1) + \beta_3^*(s_1)], [\gamma_2^*(s_1 \to s_0) + \beta_3^*(s_0)] \right) = 0.84$.

$\beta_1^*(s_0) = \max\left([\gamma_1^*(s_0 \to s_1) + \beta_2^*(s_1)], [\gamma_1^*(s_0 \to s_0) + \beta_2^*(s_0)] \right) = 1.193$

$\beta_1^*(s_1) = \max\left([\gamma_1^*(s_1 \to s_1) + \beta_2^*(s_1)], [\gamma_1^*(s_1 \to s_0) + \beta_2^*(s_0)] \right) = 2.969$

$L(u_0) = [\beta_1^*(s_1) + \gamma_0^*(s_0 \to s_1) + \alpha_0^*(s_0)] - [\beta_1^*(s_0) + \gamma_0^*(s_0 \to s_0) + \alpha_0^*(s_0)] = 1.776 > 0$

$L(u_1) = \max\left([\beta_2^*(s_0) + \gamma_1^*(s_1 \to s_0) + \alpha_1^*(s_1)], [\beta_2^*(s_1) + \gamma_1^*(s_0 \to s_1) + \alpha_1^*(s_0)] \right) - \max\left([\beta_2^*(s_0) + \gamma_1^*(s_0 \to s_0) + \alpha_1^*(s_0)], [\beta_2^*(s_1) + \gamma_1^*(s_1 \to s_1) + \alpha_1^*(s_1)] \right)$

$= 2.2 > 0$

$L(u_2) = \max\left([\beta_3^*(s_0) + \gamma_2^*(s_1 \to s_0) + \alpha_2^*(s_1)], [\beta_3^*(s_1) + \gamma_2^*(s_0 \to s_1) + \alpha_2^*(s_0)] \right) - \max\left([\beta_3^*(s_0) + \gamma_2^*(s_0 \to s_0) + \alpha_2^*(s_0)], [\beta_3^*(s_1) + \gamma_2^*(s_1 \to s_1) + \alpha_2^*(s_1)] \right)$

$= 0.9968 > 0$

$L(u_3) = [\beta_4^*(s_0) + \gamma_3^*(s_1 \to s_0) + \alpha_3^*(s_1)] - [\beta_4^*(s_0) + \gamma_3^*(s_0 \to s_0) + \alpha_3^*(s_0)] = [0 + (-0.7) + 3.582] - [0 + (-0.9) + 2.52] = 1.262 > 0$

$\therefore \hat{u} = (1, 1, 1, 1)$

Max - Log - MAP :

~~γ: the same as (b).~~

$\alpha_1^*(s_1) = \alpha_1^*(s_0) = 0$.

$\alpha_2^*(s_0) = 2.9$, $\alpha_2^*(s_1) = -0.7$.

$\alpha_3^*(s_0) = 2.52$, $\alpha_3^*(s_1) = 3.582$

$\cancel{\alpha_4^*(s_0) = 3.582}$.

$\beta_3^*(s_0) = -0.9$, $\beta_3^*(s_1) = -0.7$

$\beta_2^*(s_0) = -0.2$, $\beta_2^*(s_1) = 0.8$

$\beta_1^*(s_0) = 0.76$, $\beta_1^*(s_1) = 2.96$.

$L(u_0) = (0 + 0 + 2.96) - (0 + 0 + 0.76) = 2.2 > 0$

$L(u_1) = \max\left([0 + 2.9 - 0.2], [0 - 0.7 + 0.8] \right) - \max\left([0 + 0.7 - 0.2], [0 - 2.9 + 0.8] \right) = 2.2 > 0$

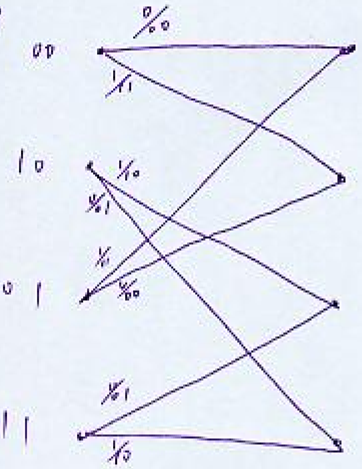
$L(u_2) = \max\left([2.9 + 0.5 - 0.7], [-0.7 - 0.5 - 0.9] \right) - \max\left([2.9 - 0.5 - 0.9], [-0.7 + 1.5 - 0.7] \right) = 1.2 > 0$

$L(u_3) = [0 + (-0.7) + 3.51] - [0 + (-0.9) + 2.52] = 2.462 > 0$

$\hat{u} = (1, 1, 1, 1)$

2.

(a)



(b) $\gamma_t'(l',l) = P_t(i)\exp\left(-\dfrac{\sum_{j=0}^{n-1}(y_{t,j}^i - x_{t,j}^i(l))^2}{2\sigma^2}\right)$

$\Rightarrow \gamma_{t-2}(0,0) = 0.5\times0.1\times0.4 = 0.02$

$\gamma_{t-2}(1,0) = 0.5\times0.5\times0.25 = 0.0625$

$\gamma_{t-2}(2,1) = 0.5\times0.5\times0.25 = 0.0625$

$\gamma_{t-2}(3,1) = 0.5\times0.1\times0.4 = 0.02$

$\gamma_{t-2}(0,2) = 0.5\times0.5\times0.25 = 0.0625$

$\gamma_{t-2}(1,2) = 0.5\times0.1\times0.4 = 0.02$

$\gamma_{t-2}(2,3) = 0.5\times0.1\times0.4 = 0.02$

$\gamma_{t-2}(3,3) = 0.5\times0.5\times0.25 = 0.0625$

$\alpha_{t-1}(0) = \alpha_{t-2}(0)\cdot\gamma_{t-2}(0,0) + \alpha_{t-2}(1)\cdot\gamma_{t-2}(1,0) = 0.5\times0.02 + 0.2\times0.0625 = 0.0225$

$\alpha_{t-1}(1) = \alpha_{t-2}(2)\cdot\gamma_{t-2}(2,1) + \alpha_{t-2}(3)\cdot\gamma_{t-2}(3,1) = 0.25\times0.0625 + 0.4\times0.02 = 0.023625$

$\alpha_{t-1}(2) = \alpha_{t-2}(0)\cdot\gamma_{t-2}(0,2) + \alpha_{t-2}(1)\cdot\gamma_{t-2}(1,2) = 0.5\times0.0625 + 0.2\times0.02 = 0.03525$

$\alpha_{t-1}(3) = \alpha_{t-2}(2)\cdot\gamma_{t-2}(2,3) + \alpha_{t-2}(3)\cdot\gamma_{t-2}(3,3) = 0.25\times0.02 + 0.4\cdot0.0625 = 0.03$.

$\gamma_t(0,0) = 0.6\times0.1\times0.4 = 0.024$

$\gamma_t(1,0) = 0.4\times0.5\times0.25 = 0.05$

$\gamma_t(2,1) = 0.4\times0.5\times0.25 = 0.05$

$\gamma_t(3,1) = 0.6\times0.1\times0.4 = 0.024$

$\gamma_t(0,2) = 0.4\times0.5\times0.25 = 0.05$

$\gamma_t(1,2) = 0.6\times0.1\times0.4 = 0.024$

$\gamma_t(2,3) = 0.6\times0.1\times0.4 = 0.024$

$\gamma_t(3,3) = 0.4\times0.5\times0.25 = 0.05$

$\gamma_{t+1}(0,0) = 0.25\times0.5\times0.25 = 0.03125$

$\gamma_{t+1}(1,0) = 0.75\times0.2\times0.3 = 0.045$

$\gamma_{t+1}(2,1) = 0.75\times0.2\times0.3 = 0.045$

$\gamma_{t+1}(3,1) = 0.25\times0.5\times0.25 = 0.03125$

$\gamma_{t+1}(0,2) = 0.75\times0.2\times0.3 = 0.045$

$\gamma_{t+1}(1,2) = 0.25\times0.5\times0.25 = 0.03125$

$\gamma_{t+1}(2,3) = 0.25\times0.5\times0.25 = 0.03125$

$\gamma_{t+1}(3,3) = 0.75\times0.2\times0.3 = 0.045$

$\beta_t(0) = \beta_{t+1}(0)\gamma_t(0,0) + \beta_{t+1}(2)\gamma_t(0,2) = 0.1\times0.024 + 0.25\times0.05 = 0.0149$

$\beta_t(1) = \beta_{t+1}(0)\gamma_t(1,0) + \beta_{t+1}(2)\gamma_t(1,2) = 0.1\times0.05 + 0.25\times0.024 = 0.011$

$\beta_t(2) = \beta_{t+1}(1)\cdot\gamma_t(2,1) + \beta_{t+1}(3)\cdot\gamma_t(2,3) = 0.5\times0.05 + 0.2\times0.024 = 0.0298$

$\beta_t(3) = \beta_{t+1}(1)\cdot\gamma_t(3,1) + \beta_{t+1}(3)\cdot\gamma_t(3,3) = 0.5\times0.024 + 0.2\times0.05 = 0.022$

$$\Lambda(c_t) = \log\frac{\sum_{(s',s)\in\Sigma_t^+}\alpha_{t-1}(l')\gamma_{t-1}(l',l)\beta_t(l)}{\sum_{(s',s)\in\Sigma_t^-}\alpha_{t-1}(l')\gamma_{t-1}(l',l)\beta_t(l)}$$

$= \log(0.0225\times0.03125\times0.0298 + 0.023625\times0.03125\times0.0149 + 0.03525\times0.03125\times0.011 + 0.03\times0.03125\times0.022)$

$-\log(0.0225\times0.045\times0.0149 + 0.023625\times0.045\times0.0298 + 0.03525\times0.045\times0.022 + 0.03\times0.045\times0.011)$

$= 0.143 > 0$

$\Rightarrow c_t = 1$