Solution 5

Sec3.2

3.2.2

Find the rank of the following matrices

(b) $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ (f) $\begin{pmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 2 & 5 & 1 \\ -4 & -8 & 1 & -3 & 1 \end{pmatrix}$

Ans.:

(b) $\because \det\begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = -1 \neq 0 \quad \therefore \text{rank}=3$

(f) $$\because \underset{(-2)}{\begin{pmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 2 & 5 & 1 \\ -4 & -8 & 1 & -3 & 1 \end{pmatrix}} \rightarrow \underset{(-2)\quad(-3)}{\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 & 1 \\ -4 & 0 & 1 & -3 & 1 \end{pmatrix}} \rightarrow \underset{(-1)}{\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 & 1 \\ -6 & 0 & 1 & -6 & 1 \end{pmatrix}} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 \\ -6 & 0 & 1 & 0 & 1 \end{pmatrix} \quad \therefore \text{rank}=3$$

3.2.5

For each of the following matrices, compute the rank and the inverse if it exists.

(g) $\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{pmatrix}$

Ans.:

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 5 & 5 & 1 & 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 3 & 0 & 0 & 1 & 0 \\ 3 & 4 & -2 & -3 & 0 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & 2 & 0 & 1 & 0 \\ 0 & -2 & -5 & -3 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow\left(\begin{array}{cccc|cccc}1&0&-5&-2&5&-2&0&0\\0&1&3&1&-2&1&0&0\\0&0&-1&2&4&-1&1&0\\0&0&1&1&7&2&0&1\end{array}\right)\Rightarrow\left(\begin{array}{cccc|cccc}1&0&0&-12&-15&3&-5&0\\0&1&0&7&10&-2&3&0\\0&0&1&-2&-4&1&-1&0\\0&0&0&1&-3&1&1&1\end{array}\right)$$

$$\Rightarrow\left(\begin{array}{cccc|cccc}1&0&0&0&-51&15&7&12\\0&1&0&0&31&-9&-4&-7\\0&0&1&0&-10&3&1&2\\0&0&0&1&-3&1&1&1\end{array}\right)\ \therefore\text{rank}=4$$

Inverse matrix is $\begin{pmatrix}-51&15&7&12\\31&-9&-4&-7\\-10&3&1&2\\-3&1&1&1\end{pmatrix}$

3.2.7

Express the invertible matrix

$$\begin{pmatrix}1&2&1\\1&0&1\\1&1&2\end{pmatrix}$$

As a product of elementary matrices.

Ans.:

$$\begin{pmatrix}1&2&1\\1&0&1\\1&1&2\end{pmatrix}=\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\1&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&-2&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&2&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&-1&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$$

Sec3.3

3.3.2

For each of the following homogeneous systems of linear equation, find the dimension of and a basis for the solution set.

(c) $\begin{array}{l}x_1+2x_2-x_3=0\\2x_1+x_2+x_3=0\end{array}$ (d) $\begin{array}{l}2x_1+x_2-x_3=0\\x_1-x_2+x_3=0\\x_1+2x_2-2x_3=0\end{array}$

Ans.:

(c) $x_1=-x_2=-x_3\Rightarrow$ dim=1, Basis is $\left\{\begin{pmatrix}1\\-1\\-1\end{pmatrix}\right\}$

(d) $x_1=0$, $x_2=x_3 \Rightarrow$ dim=1, Basis is $\left\{\begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

3.3.3

(c) $\begin{matrix} x_1+2x_2-x_3=3 \\ 2x_1+x_2+x_3=6 \end{matrix}$ (d) $\begin{matrix} 2x_1+x_2-x_3=5 \\ x_1-x_2+x_3=1 \\ x_1+2x_2-2x_3=4 \end{matrix}$

Ans.:

(c) $3x_1+3x_2=9$, $x_1+x_3=3$, let $x_1=1$, $x_3=2$ $\Rightarrow x_2=2$

$$\Rightarrow \left\{\begin{pmatrix}1\\2\\2\end{pmatrix}+t\begin{pmatrix}0\\1\\1\end{pmatrix},\ t\in R\right\}$$

(d) $\left\{\begin{pmatrix}2\\1\\0\end{pmatrix}+t\begin{pmatrix}0\\1\\1\end{pmatrix},\ t\in R\right\}$

3.3.7

Determine which of the following systems of linear equations has a solution.

(b) $\begin{matrix} x_1+x_2-x_3=1 \\ 2x_1+x_2+3x_3=2 \end{matrix}$ (c) $\begin{matrix} x_1+2x_2+3x_3=1 \\ x_1+x_2-x_3=0 \\ x_1+2x_2+x_3=3 \end{matrix}$

Ans.:

(b) linear independent $\Rightarrow$ has a sulotion

(c) $A=\begin{pmatrix}1&2&3\\1&1&-1\\1&2&1\end{pmatrix}$, $A|b=\begin{pmatrix}1&2&3&1\\1&1&-1&0\\1&2&1&3\end{pmatrix}$

$\because$rank(A)= rank(A|b)=3 $\therefore$has a sulotion

Sec3.4

3.4.1

Label the following statements as true or false.

(b) If $(A'|b')$ is obtained from $(A|b)$ by a finite sequence of elementary row operations, then the systems $Ax=b$ and $A'x=b'$ are equivalent.

Ans.:

(b) True.

3.4.2

Use Gaussian elimination to solve the following systems of linear equations.

(b) $\begin{array}{l} x_1 - 2x_2 - x_3 = 1 \\ 2x_1 - 3x_2 + x_3 = 6 \\ 3x_1 - 5x_2 = 7 \\ x_1 + 5x_3 = 9 \end{array}$

(c) $\begin{array}{l} x_1 + 2x_2 + 2x_4 = 6 \\ 3x_1 + 5x_2 - x_3 + 6x_4 = 17 \\ 2x_1 + 4x_2 + x_3 + 2x_4 = 12 \\ 2x_1 - 7x_3 + 11x_4 = 7 \end{array}$

Ans.:

(b) $\because (x_1 - 2x_2 - x_3 = 1) \oplus (2x_1 - 3x_2 + x_3 = 6) = (3x_1 - 5x_2 = 7)$

$$\to \begin{pmatrix} 1 & -2 & -1 & 1 \\ 2 & -3 & 1 & 6 \\ 1 & 0 & 5 & 9 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 6 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 5 & 9 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 5x_3 = 9 \\ x_2 + 3x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 = -5x_3 \\ x_2 = -3x_3 \end{cases}$$

$$\Rightarrow \left\{ \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ -3 \\ 1 \end{pmatrix}, t \in R \right\}$$

(c) $$\begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 17 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -7 & 11 & 7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & 7 & 7 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -4 & 7 & 7 & -5 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -2 & 2 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 7 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -2 & 4 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \to \left\{ \begin{pmatrix} 2 \\ 3 \\ -2 \\ -1 \end{pmatrix} \right\}$$

3.4.4

For each of the systems that follow, apply Exercise 3 to determine whether the system is consistent. If the system is consistent, find all solutions. Finally, find a basis for the solution set of the corresponding homogeneous system.

$$x_1 + 2x_2 - x_3 + x_4 = 2$$

(a) $$2x_1 + x_2 + x_3 - x_4 = 3$$

$$x_1 + 2x_2 - 3x_3 + 2x_4 = 2$$

Ans.:

$$\begin{pmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 1 & 1 & -1 & 3 \\ 1 & 2 & -3 & 2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & -3 & 3 & -3 & -1 \\ 0 & 0 & -2 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & -1 & 1 & 1/3 \\ 0 & 0 & 1 & -1/2 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 1 & -1 & 4/3 \\ 0 & 1 & -1 & 1 & 1/3 \\ 0 & 0 & 1 & -1/2 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -1/2 & 4/3 \\ 0 & 1 & 0 & 1/2 & 1/3 \\ 0 & 0 & 1 & -1/2 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 - \frac{1}{2}x_4 &= \frac{4}{3} \\ x_2 + \frac{1}{2}x_4 &= \frac{1}{3} \\ x_3 - \frac{1}{2}x_4 &= 0 \end{aligned} \to \left\{ \begin{pmatrix} \frac{4}{3} \\ \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \end{pmatrix}, t \in R \right\}$$

The basis of Homogeneous part: $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \end{pmatrix} \right\}$