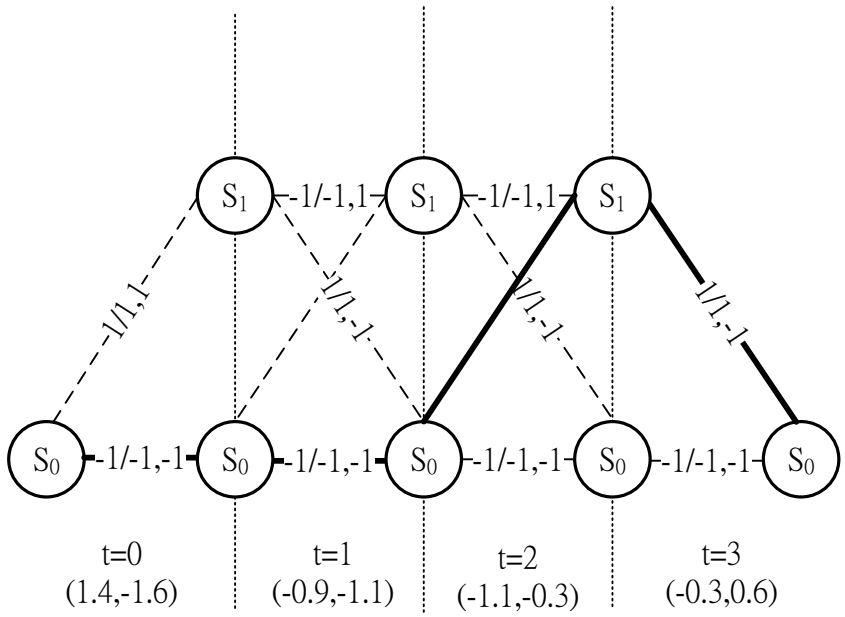
2008 Advanced Coding Theorem HW#1 solution

1.



(a) SOVA:

(1) t=0

$S_1 : M^*([r \mid V=1]_1) = 0+2*[1*1.4+1*(-1.6)]=-0.4$

$S_0 : M^*([r \mid V=-1]_1) = 0+2*[(-1)*1.4+(-1)*(-1.6)]=0.4$

(2) t=1

$$S_1 : M^*([r \mid V=1]_2) = 0.4+2*[1*(-0.9)+1*(-1.1)] = -3.6$$
$$M^*([r \mid V=-1]_2) = -0.4+2*[(-1)*(-0.9)+(1)*(-1.1)] = -0.8$$

$$S_0 : M^*([r \mid V=1]_2) = 0.4+2*(0.9+1.1) = 4.4$$
$$M^*([r \mid V=-1]_2) = -0.4+2*(-0.9+1.1) = 0$$

Survivor of S_1 : -0.8

Survivor of S_0 : 4.4

(3) t=2

$$S_1 : M^*([r \mid V=1]_3) = -0.8+2*(0.3+1.1) = 2$$
$$M^*([r \mid V=-1]_3) = 4.4+2*(-0.3+1.1) = 6$$

$$S_0 : M^*([r \mid V=1]_3) = -0.8+2*(-0.3-1.1) = -3.6$$
$$M^*([r \mid V=-1]_3) = 4.4+2*(0.3-1.1) = 2.8$$

Survivor of S_1 : 6

Survivor of S_0 : 2.8

(4) t=3

$$S_1 : M^*([r \mid V=1]_4) = 2.8 + 2*(0.3-0.6) = 2.2$$
$$M^*([r \mid V=-1]_4) = 6 + 2*(-0.3-0.6) = 4.2$$

Survivor of S_1 : 4.2

From survivor path, we can decode : (u_0, u_1, u_2, u_3)=(-1,-1,1,1)

$$L(u_l) \cong \hat{u}_l \min_{i=l,\ldots l+\delta} \Delta_i \Rightarrow$$

$$\Delta_3 = 0.5*(4.2-2.2) = 1,$$
$$\Delta_2 = 0.5*(6-2) = 2$$
$$\Delta_1 = 0.5*(4.4-0) = 2.2,$$
$$\Delta_0 = \mathrm{X}$$
$$L(u_0) = -1*\min(\Delta_0, \Delta_1, \Delta_2, \Delta_3) = -1$$
$$L(u_1) = -1*\min(\Delta_1, \Delta_2, \Delta_3) = -1$$
$$L(u_2) = 1*\min(\Delta_2, \Delta_3) = 1$$
$$L(u_3) = 1*\min(\Delta_3) = 1$$

(b) Max-log-MAP:

$$\gamma_0^*(s_0, s_0) = 0.2$$
$$\gamma_0^*(s_0, s_1) = -0.2$$
$$\gamma_1^*(s_0, s_0) = 2$$
$$\gamma_1^*(s_0, s_1) = -2$$
$$\gamma_1^*(s_1, s_0) = 0.2$$
$$\gamma_1^*(s_1, s_1) = -0.2$$
$$\gamma_2^*(s_0, s_0) = -0.8$$
$$\gamma_2^*(s_0, s_1) = 0.8$$
$$\gamma_2^*(s_1, s_0) = -1.4$$
$$\gamma_2^*(s_1, s_1) = 1.4$$
$$\gamma_3^*(s_0, s_0) = -0.3$$
$$\gamma_3^*(s_1, s_0) = -0.9$$

$\alpha_0^*(s_0) = 0$
$\alpha_1^*(s_0) = 0.2$
$\alpha_1^*(s_1) = -0.2$
$\alpha_2^*(s_0) = 2.2$
$\alpha_2^*(s_1) = 0$
$\alpha_3^*(s_0) = 1.4$
$\alpha_3^*(s_1) = 3$
$\beta_4^*(s_0) = 0$
$\beta_3^*(s_0) = -0.3$
$\beta_3^*(s_1) = -0.9$
$\beta_2^*(s_0) = -0.1$
$\beta_2^*(s_1) = 0.5$
$\beta_1^*(s_0) = 1.9$
$\beta_1^*(s_1) = 0.3$

$L(u_0) = (-0.2 + 0.3) - (1.9 + 0.2) = -2$
$L(u_0) = -0.1 - 2.1 = -2.2$
$L(u_0) = 2.1 - 1.1 = 1$
$L(u_0) = 2.1 - 1.1 = 1$
$\Rightarrow \hat{u}_l = (-1,-1,1,1)$

(c) Log-MAP:

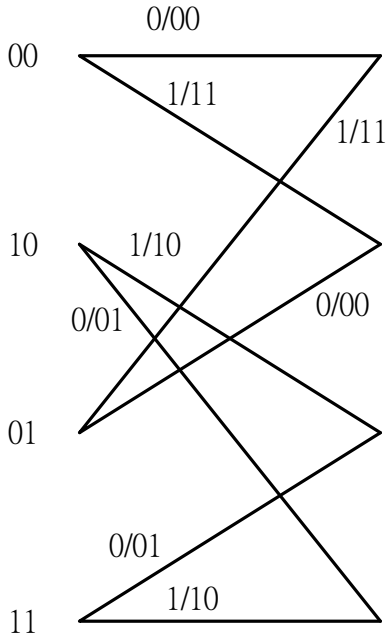
Using the max calculates function: $\max^*(x, y) = \ln(e^x + e^y) = \max(x, y) + \ln(1 + e^{-|x-y|})$

$\alpha_0^*(s_0) = 0$
$\alpha_1^*(s_0) = 0.2$
$\alpha_1^*(s_1) = -0.2$
$\alpha_2^*(s_0) = \max^*(0,2.2) = 2.3$
$\alpha_2^*(s_1) = \max^*(-0.4,-1.8) = -0.18$
$\alpha_3^*(s_0) = \max^*(-1.8,1.4) = 1.44$
$\alpha_3^*(s_1) = \max^*(1,3) = 3.1$

$\beta_4^*(s_0) = 0$
$\beta_3^*(s_0) = -0.3$
$\beta_3^*(s_1) = -0.9$
$\beta_2^*(s_0) = \max^*(0.5,-1.7) = 0.6$
$\beta_2^*(s_1) = \max^*(-0.1,-1.2) = 0.19$
$\beta_1^*(s_0) = \max^*(0.3,0.1) = 0.89$
$\beta_1^*(s_1) = \max^*(-1.5,1.9) = 1.9$

$L(u_0) = (-0.2 + 0.89) - (0.2 + 1.9) = -1.41$
$L(u_1) = \max^*(-0.2 + 0.2 + 0.19, 0.2 - 2 + 0.6) - \max^*(-0.2 - 0.2 + 0.6, 0.2 + 2 + 0.19) = -2.09$
$L(u_2) = \max^*(-0.18 - 1.4 - 0.3, 2.3 + 0.8 - 0.9) - \max^*(-0.18 + 1.4 - 0.9, 0.3 - 0.8 - 0.3) = 0.75$
$L(u_3) = (3.1 - 0.9) - (1.44 - 0.3) = 1.06 \Rightarrow u = [0,0,1,1]$

2.



$$\gamma_t^{'}(l',l) = P_t(i)\exp\left(-\frac{\sum_{j=0}^{m-1}(\gamma_{t,j}^{i} - x_{t,j}^{i}(l))^2}{2\sigma^2}\right)$$

$\gamma_{t-2}(0,0) = 0.5*0.1*0.4 = 0.02$
$\gamma_{t-2}(1,0) = 0.5*0.5*0.25 = 0.0625$
$\gamma_{t-2}(2,1) = 0.5*0.5*0.4 = 0.1$
$\gamma_{t-2}(3,1) = 0.5*0.1*0.25 = 0.0125$
$\gamma_{t-2}(0,2) = 0.5*0.5*0.25 = 0.0625$
$\gamma_{t-2}(1,2) = 0.5*0.1*0.4 = 0.02$
$\gamma_{t-2}(2,3) = 0.5*0.1*0.25 = 0.0125$
$\gamma_{t-2}(3,3) = 0.5*0.5*0.4 = 0.1$

$\alpha_{t-1}(0) = \alpha_{t-2}(0)*\gamma_{t-2}(0,0) + \alpha_{t-2}(1)*\gamma_{t-2}(1,0) = 0.5*0.02 + 0.2*0.0625 = 0.0225$
$\alpha_{t-1}(1) = \alpha_{t-2}(2)*\gamma_{t-2}(2,1) + \alpha_{t-2}(3)*\gamma_{t-2}(3,1) = 0.25*0.0625 + 0.4*0.02 = 0.023625$
$\alpha_{t-1}(2) = \alpha_{t-2}(0)*\gamma_{t-2}(0,2) + \alpha_{t-2}(1)*\gamma_{t-2}(1,2) = 0.5*0.0625 + 0.2*0.02 = 0.03525$
$\alpha_{t-1}(3) = \alpha_{t-2}(2)*\gamma_{t-2}(2,3) + \alpha_{t-2}(3)*\gamma_{t-2}(3,3) = 0.25*0.02 + 0.4*0.0625 = 0.03$

$$\gamma_{t-1}(0,0) = 0.25*0.25*0.5 = 0.03125$$
$$\gamma_{t-1}(1,0) = 0.75*0.2*0.3 = 0.045$$
$$\gamma_{t-1}(2,1) = 0.75*0.2*0.5 = 0.075$$
$$\gamma_{t-1}(3,1) = 0.25*0.25*0.3 = 0.01875$$
$$\gamma_{t-1}(0,2) = 0.75*0.2*0.3 = 0.045$$
$$\gamma_{t-1}(1,2) = 0.25*0.25*0.5 = 0.03125$$
$$\gamma_{t-1}(2,3) = 0.25*0.25*0.3 = 0.01875$$
$$\gamma_{t-1}(3,3) = 0.75*0.2*0.5 = 0.075$$
$$\gamma_{t}(0,0) = 0.6*0.4*0.1 = 0.024$$
$$\gamma_{t}(1,0) = 0.4*0.25*0.5 = 0.05$$
$$\gamma_{t}(2,1) = 0.4*0.25*0.1 = 0.01$$
$$\gamma_{t}(3,1) = 0.6*0.4*0.5 = 0.12$$
$$\gamma_{t}(0,2) = 0.4*0.25*0.5 = 0.05$$
$$\gamma_{t}(1,2) = 0.6*0.4*0.1 = 0.024$$
$$\gamma_{t}(2,3) = 0.6*0.4*0.5 = 0.12$$
$$\gamma_{t}(3,3) = 0.4*0.25*0.1 = 0.01$$

$$\beta_t(0) = \beta_{t+1}(0)*\gamma_t(0,0) + \beta_{t+1}(2)*\gamma_t(0,2) = 0.1*0.024 + 0.25*0.05 = 0.0149$$
$$\beta_t(1) = \beta_{t+1}(0)*\gamma_t(1,0) + \beta_{t+1}(2)*\gamma_t(1,2) = 0.1*0.05 + 0.25*0.024 = 0.011$$
$$\beta_t(2) = \beta_{t+1}(1)*\gamma_t(2,1) + \beta_{t+1}(3)*\gamma_t(2,3) = 0.5*0.01 + 0.2*0.12 = 0.029$$
$$\beta_t(3) = \beta_{t+1}(1)*\gamma_t(3,1) + \beta_{t+1}(3)*\gamma_t(3,3) = 0.5*0.12 + 0.2*0.01 = 0.062$$

$$\Lambda(c_t) = \ln \frac{\sum_{(s',s)\in \sum l^{+}} \alpha_{t-1}(l^{'}) \cdot \gamma_{t-1}(l',l) \cdot \beta_t(0)}{\sum_{(s',s)\in \sum l^{-}} \alpha_{t-1}(l^{'}) \cdot \gamma_{t-1}(l',l) \cdot \beta_t(0)}$$

$$= \ln \frac{\alpha_{t-1}(0) \cdot \gamma_{t-1}(0,2) \cdot \beta_t(2) + \alpha_{t-1}(2) \cdot \gamma_{t-1}(2,1) \cdot \beta_t(1) + \alpha_{t-1}(1) \cdot \gamma_{t-1}(1,0) \cdot \beta_t(0) + \alpha_{t-1}(3) \cdot \gamma_{t-1}(3,3) \cdot \beta_t(3)}{\alpha_{t-1}(0) \cdot \gamma_{t-1}(0,0) \cdot \beta_t(0) + \alpha_{t-1}(2) \cdot \gamma_{t-1}(2,3) \cdot \beta_t(3) + \alpha_{t-1}(1) \cdot \gamma_{t-1}(1,2) \cdot \beta_t(2) + \alpha_{t-1}(3) \cdot \gamma_{t-1}(3,1) \cdot \beta_t(1)}$$

$$= \ln \frac{0.0225*0.045*0.029 + 0.03525*0.075*0.011 + 0.023625*0.045*0.0149 + 0.03*0.075*0.062}{0.0225*0.03125*0.0149 + 0.03525*0.01875*0.062 + 0.023625*0.03125*0.029 + 0.03*0.01875*0.011}$$

$$= 0.994857 > 0 \Rightarrow c_t = 1$$