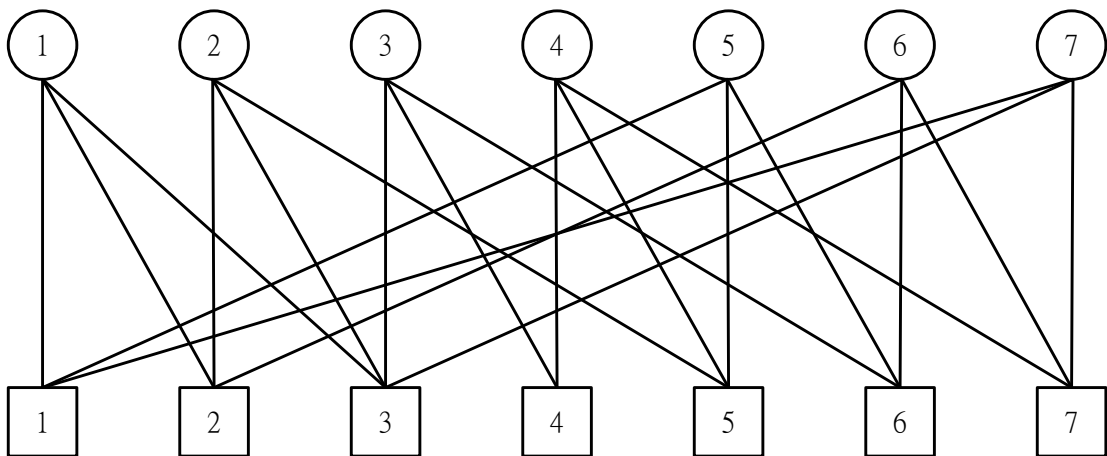


## 2008 Advanced Coding Theorem HW#2 solution

$$1. \text{ (a) } \because \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \therefore d_{\min} = 4$$

(b)



(c) No

$$h_{11} \rightarrow h_{12} \rightarrow h_{62} \rightarrow h_{66} \rightarrow h_{56} \rightarrow h_{51} \rightarrow h_{11}$$

$$h_{11} \rightarrow h_{12} \rightarrow h_{22} \rightarrow h_{25} \rightarrow h_{55} \rightarrow h_{51} \rightarrow h_{11}$$

$$h_{11} \rightarrow h_{12} \rightarrow h_{62} \rightarrow h_{67} \rightarrow h_{77} \rightarrow h_{71} \rightarrow h_{11}$$

$$(d) h_{11} \rightarrow h_{14} \rightarrow h_{34} \rightarrow h_{33} \rightarrow h_{73} \rightarrow h_{71} \rightarrow h_{11}$$

$$h_{11} \rightarrow h_{14} \rightarrow h_{34} \rightarrow h_{36} \rightarrow h_{56} \rightarrow h_{51} \rightarrow h_{11}$$

$$h_{11} \rightarrow h_{14} \rightarrow h_{44} \rightarrow h_{45} \rightarrow h_{55} \rightarrow h_{51} \rightarrow h_{11}$$

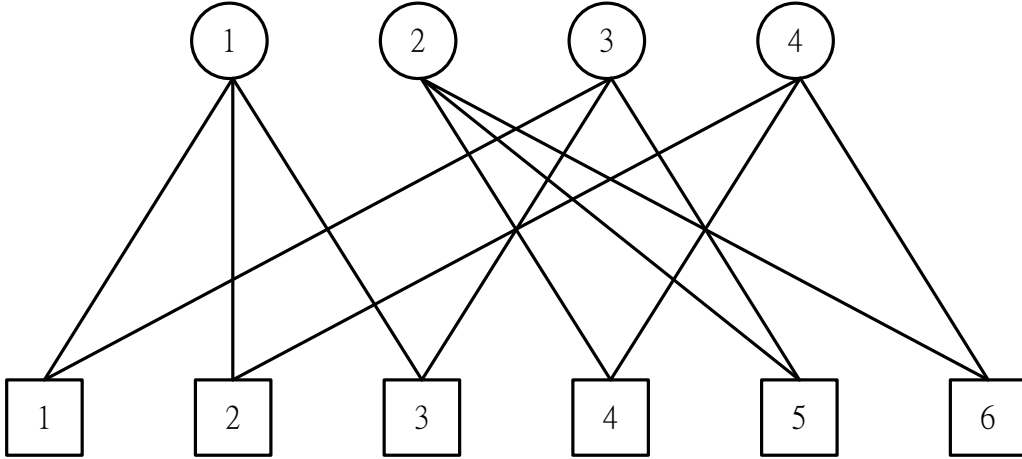
$$h_{11} \rightarrow h_{14} \rightarrow h_{44} \rightarrow h_{47} \rightarrow h_{77} \rightarrow h_{71} \rightarrow h_{11}$$

There are 7 branch starting from and back to  $h_{11}$ .

Because  $i$ th row in H is shift by  $(i-1)$ th. We can know total number of cycle 6 is

$$7 \cdot 7 = 49$$

2.



$$\begin{aligned} \mu_{C \rightarrow V_1}^{(1)} &= \mu_{V_1}^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)} \right] = 1.115996 \\ \mu_{C \rightarrow V_2}^{(1)} &= \mu_2^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)} \right] = -0.74657 \\ \mu_{C \rightarrow V_3}^{(1)} &= \mu_3^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)} \right] = -0.88077 \\ \mu_{C \rightarrow V_4}^{(0)} &= \mu_4^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)} \right] = 0.165159 \\ \mu_{C \rightarrow V_5}^{(1)} &= \mu_{V_5}^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_6}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_1}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_3}^{(0)}}{2}\right)} \right] = -0.94657 \\ \mu_{C \rightarrow V_6}^{(1)} &= \mu_{V_6}^{(0)} + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_5}^{(0)}}{2}\right)} \right] + \log \left[ \frac{1 + \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right)}{1 - \tanh\left(\frac{\mu_{V_2}^{(0)}}{2}\right) \cdot \tanh\left(\frac{\mu_{V_4}^{(0)}}{2}\right)} \right] = -0.216 \end{aligned}$$

Do hard decision and check  $c\mathbf{H}^t$  is zero or not.

Because  $c\mathbf{H}^t$  is zero => finish.

Hard decision: {0,1,1,0,1,1}